# Vibration Monitoring by Eigenstructure Change Detection Based on Perturbation Analysis

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**Abstract:** Vibration monitoring, notably in the fields of civil, mechanical and aeronautical engineering, aims at detecting damages at an early stage, in general by using output-only vibration measurements under ambient excitation. In this paper, a new method is proposed for the detection of small changes in the eigenstructure of such systems. The main idea is to transform the multiplicative eigenstructure change detection problem to an additive one, by means of perturbation analysis based on the assumption of small eigenstructure changes. Another transformation then further simplifies the detection problem into the framework of a linear regression subject to additive white Gaussian noises, leading to a numerically efficient solution of the considered problem. Compared to existing methods, it has the advantages of focusing on chosen system parameters and efficiently addressing random uncertainties. A numerical example of a simulated mechanical structure and a lab experiment on a beam, each with the detection of different damages, are reported.

Keywords: Vibration monitoring, eigenstructure, statistical tests, fault detection.

## 1. INTRODUCTION

The detection of damages based on measured vibration data is a fundamental task for structural health monitoring to allow an automated damage diagnosis, e.g., for mechanical, civil or aeronautical structures [Farrar and Worden, 2007]. If damages are detected early enough, they can be accommodated by maintaining the structures in appropriate operational states. Damage affects the dynamic properties of a structure, inducing changes in the modal parameters (natural frequencies, damping ratios, mode shapes), or in their equivalent eigenstructure representation (eigenvalues and observed eigenvectors). Damage detection thus amounts to detecting *small changes* in the eigenstructure of the monitored structure. A particular difficulty for structural health monitoring is caused by the absence of known system inputs, since the structural excitation is usually only ambient, leading to an output-only monitoring problem.

There exist various methods for mechanical structure damage detection [Carden and Fanning, 2004], either model-based or data-based. Model-based detection methods typically fall into two categories, respectively based on repeated system identification and on direct model-data matching.

Within the methods based on repeated system identification, a common strategy is to repeatedly estimate current modal parameters by means of system identification, and to compare the result to some reference modal parameters [Magalhães et al., 2012]. Tracking the evolution of parameters over time, recursive identification schemes may be used, e.g., based on subspace methods [Goethals et al., 2004], on maximum likelihood estimation using a Kalman filter coupled with a tangent filter [Campillo and Mevel, 2005] or on interacting Kalman filter and particle filter [Zghal et al., 2014].

With the methods based on direct model-data matching, current measurement data are directly confronted to a reference model, without resorting to repeated system identification. For instance, such methods include non-parametric change detection based on novelty detection [Worden et al., 2000, Yan et al., 2004] or whiteness tests on Kalman filter innovations [Bernal, 2013]. Another method within this category, the local asymptotic approach to change detection [Benveniste et al., 1987], has the ability of focusing the detection on some chosen system parameters, in particular on modal parameters of mechanical structures. Associated to efficient hypothesis testing tools, this method has led to successful applications in the field of vibration monitoring (Basseville et al. [2000], Zouari et al. [2009], Jhinaoui et al. [2012], Döhler and Mevel [2013], Döhler et al. [2014a,b]).

The purpose of the present paper is to propose another method based on direct model-data matching, which can also focus the detection on chosen system parameters, and has the advantage of more efficiently addressing random uncertainties when associated to hypothesis testing tools. In this method, the normalization of the involved statistical tests is fully based on the covariance matrices of the state and output noises estimated at the step of the nominal system identification, whereas in the local asymptotic

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approach to change detection [Benveniste et al., 1987], the statistical tests are normalized by the covariance matrix of the sum of some designed residual vector, whose estimation requires large data samples.

The eigenstructure change detection problem is *multiplicative* in the sense that the eigenstructure parameters appear in coefficients of the unknown state vector of the state space model characterizing mechanical structures. The main idea of the present paper is to transform this problem to an *additive* change detection problem, by applying the perturbation analysis based on the assumption of small eigenstructure changes. After this transformation, another step based on the results of (Zhang and Basseville [2014]) then further transforms the problem into the form of a linear regression with additive Gaussian noises. The covariance matrix of these Gaussian noises is easily determined from the properties of the nominal state space model of the monitored system, typically obtained by means of system identification.

This paper is organized as follows. In Section 2, the structural modelization is recalled and the fault detection problem is stated. In Section 3, the perturbation analysis is carried out to transform this problem into an additive one, and in Section 4 the respective hypothesis test is derived. An algorithmic summary of the developed method is given in Section 5. Finally, applications for vibration-based damage detection are shown in Section 6 in simulations and on a lab structure.

## 2. PROBLEM STATEMENT

The behavior of linear time-invariant mechanical structures subject to unknown ambient excitation can be described by the differential equation

$$\mathcal{M}\ddot{\mathcal{X}}(t) + \mathcal{C}\dot{\mathcal{X}}(t) + \mathcal{K}\mathcal{X}(t) = \upsilon(t) \tag{1}$$

where $t$ denotes continuous time; $\mathcal{M}, \mathcal{C}, \mathcal{K} \in \mathbb{R}^{m\times m}$ are mass, damping, and stiffness matrices, respectively; the state vector $\mathcal{X}(t) \in \mathbb{R}^m$ is the displacement vector of the $m$ degrees of freedom of the structure; and $\upsilon(t)$ is the external unmeasured force (noise).

Observed at $r$ sensor positions (e.g. displacement, velocity or acceleration sensors) at discrete time instants $t = k\tau$ (with sampling rate $1/\tau$), system (1) can also be described by a discrete-time state space system model [Juang, 1994]

$$\begin{cases} z_{k+1} = Fz_k + \omega_k \\ y_k = Hz_k + v_k \end{cases} \tag{2}$$

where the state vector $z_k = \left[\mathcal{X}(k\tau)^T \ \dot{\mathcal{X}}(k\tau)^T\right]^T \in \mathbb{R}^n$ with $n = 2m$, the measured output vector $y_k \in \mathbb{R}^r$, the system matrices

$$F = \exp\left(\begin{bmatrix} 0 & I \\ -\mathcal{M}^{-1}\mathcal{K} & -\mathcal{M}^{-1}\mathcal{C} \end{bmatrix}\tau\right) \in \mathbb{R}^{n\times n}$$
$$H = \begin{bmatrix} L_d - L_a\mathcal{M}^{-1}\mathcal{K} & L_v - L_a\mathcal{M}^{-1}\mathcal{C} \end{bmatrix} \in \mathbb{R}^{r\times n},$$

with selection matrices $L_d, L_v, L_c \in \{0,1\}^{r\times m}$ indicating the positions of displacement, velocity or acceleration sensors. The state noise $\omega_k$ and output noise $v_k$ are unmeasured and assumed to be Gaussian, zero-mean, white.

Damage leads to changes in the structural properties of system (1), e.g., in mass parameters of elements in $\mathcal{M}$, or in parameters corresponding to element stiffness such as Young's modulus in $\mathcal{K}$. Hence they provoke changes in the eigenstructure of system (1), and consequently of system (2), which shall be monitored for changes. The eigenstructure of (2) is defined as the collection of eigenvalues and observed eigenvectors $(\lambda_i, \varphi_i)$ with

$$F\phi_i = \lambda_i\phi_i, \ \varphi_i = H\phi_i, \quad i = 1, 2, \ldots, n \tag{3}$$

and constitutes a canonical parameterization invariant to linear state transformations.

Assume that the eigenstructure of the considered system contains only complex modes. This is the typical case for structural health monitoring applications. Then, the eigenvalues $\lambda_i$ of the real matrix $F \in \mathbb{R}^{n\times n}$ $(n = 2m)$ consist of $m$ conjugate complex pairs. Let the vector

$$\lambda \triangleq [\lambda_1, \lambda_2, \ldots, \lambda_m]^T \in \mathbb{C}^m$$

contain $m$ of the $n$ eigenvalues $\lambda_i$, one out of each of the $m$ conjugate complex pairs, and

$$\varphi \triangleq [\varphi_1, \varphi_2, \ldots, \varphi_m] \in \mathbb{C}^{r\times m}$$

be composed of the corresponding observed eigenvectors. The complex eigenstructure $(\lambda_i, \varphi_i)$ is then represented by the equivalent real parameter vector $\theta \in \mathbb{R}^{n+nr}$ defined as

$$\theta \triangleq \begin{bmatrix} \mathrm{Re}(\lambda) \\ \mathrm{Im}(\lambda) \\ \mathrm{Re}(\mathrm{vec}(\varphi)) \\ \mathrm{Im}(\mathrm{vec}(\varphi)) \end{bmatrix} \tag{4}$$

where Re and Im denote respectively the real part and the imaginary part of a complex value.

Assume that the matrix $F$ in (3) is diagonalizable. Let the real matrices $A(\theta) \in \mathbb{R}^{n\times n}, C(\theta) \in \mathbb{R}^{r\times n}$ be defined as

$$A(\theta) = \begin{bmatrix} \mathrm{Re}(\mathrm{diag}(\lambda)) & \mathrm{Im}(\mathrm{diag}(\lambda)) \\ -\mathrm{Im}(\mathrm{diag}(\lambda)) & \mathrm{Re}(\mathrm{diag}(\lambda)) \end{bmatrix} \tag{5a}$$
$$C(\theta) = [\mathrm{Re}(\varphi) \ \mathrm{Im}(\varphi)] \tag{5b}$$

then the state space system (2) is equivalent to

$$x_{k+1} = A(\theta)x_k + w_k \tag{6a}$$
$$y_k = C(\theta)x_k + v_k \tag{6b}$$

in the sense that there exists an invertible matrix $T \in \mathbb{R}^{n\times n}$ such that $x_k = Tz_k$ and $w_k = T\omega_k$. This *eigen-canonical state space model* will be used in this paper for detecting changes in the eigenstructure.

In practice, the mass, damping, stiffness matrices $\mathcal{M}, \mathcal{C}, \mathcal{K} \in \mathbb{R}^{m\times m}$ are usually unknown. To characterize the considered system, a state space model of order $n = 2m$ can be estimated from available output data $y_k$ by methods of system identification (Van Overschee and De Moor [1996], Ljung [1999]). The computation of the eigenvalues and eigenvectors of the estimated system matrix then leads to the nominal parameter vector, say

$$\theta^0 \triangleq \begin{bmatrix} \mathrm{Re}(\lambda^0) \\ \mathrm{Im}(\lambda^0) \\ \mathrm{Re}(\mathrm{vec}(\varphi^0)) \\ \mathrm{Im}(\mathrm{vec}(\varphi^0)) \end{bmatrix}, \tag{7}$$

characterizing the nominal state space model in the eigen-canonical form as defined by (5) and (6). The covariance matrices of the state and output noises,

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} = \mathrm{E}\left(\begin{bmatrix} w_k \\ v_k \end{bmatrix} \begin{bmatrix} w_k^T & v_k^T \end{bmatrix}\right), \tag{8}$$

are also derived from the result of system identification and from the transformation to the eigen-canonical form.

Assume that the currently monitored system is subject to a *small* change of the parameter vector, so that the value of $\theta$ becomes

$$\theta = \theta^0 + \varepsilon\theta^1 \tag{9}$$

where the *unknown* vector $\theta^1 \in \mathbb{R}^n$ is expressed as

$$\theta^1 = \begin{bmatrix} \mathrm{Re}(\lambda^1) \\ \mathrm{Im}(\lambda^1) \\ \mathrm{Re}(\mathrm{vec}(\varphi^1)) \\ \mathrm{Im}(\mathrm{vec}(\varphi^1)) \end{bmatrix}$$

and $\varepsilon > 0$ is a small constant reflecting the fact that the parameter increment

$$\tilde{\theta} \triangleq \varepsilon\theta^1 \tag{10}$$

is small. Under the *small* change assumption, the parameter vector $\theta$ deviated from the nominal value $\theta^0$ remains approximately an eigenstructure parameter vector.

The remaining part of this paper is for the purpose of developing a method for detecting *small* changes $\tilde{\theta} = \varepsilon\theta^1$ possibly affecting the parameter vector $\theta$.

## 3. PERTURBATION ANALYSIS

The main difficulty for detecting changes in the parameter vector $\theta$ resides in the fact that the matrices $A(\theta), C(\theta)$, possibly subject to *unknown* changes of $\theta$, appear in products with the *unknown* state vector $x_k$ in the state space model (6). The purpose of this section is to transform this *multiplicative* change detection problem into an *additive* form by means of perturbation analysis.

As the matrices $A(\theta), C(\theta)$ defined in (5) are *linearly* parametrized by the vector $\theta$ as composed in (4), the decomposition of $\theta$ in (9) leads to

$$A(\theta) = A(\theta^0) + \varepsilon A(\theta^1)$$
$$C(\theta) = C(\theta^0) + \varepsilon C(\theta^1).$$

For more compact notations, let us define

$$A^0 \triangleq A(\theta^0), \quad A^1 \triangleq A(\theta^1)$$
$$C^0 \triangleq C(\theta^0), \quad C^1 \triangleq C(\theta^1)$$

then

$$A(\theta) = A^0 + \varepsilon A^1 \tag{13a}$$
$$C(\theta) = C^0 + \varepsilon C^1. \tag{13b}$$

Based on the nominal system model characterized by $\theta^0$, the Kalman filter can compute the one-step-ahead state prediction, denoted by the short notation

$$x_k^0 \triangleq x_{k|k-1}(\theta^0).$$

Le $K$ be the steady state Kalman gain, then $x_k^0$ is recursively computed as

$$x_{k+1}^0 = A^0 x_k^0 + A^0 K(y_k - C^0 x_k^0) \tag{14a}$$
$$x_0^0 = \mathrm{E}(x_0) \tag{14b}$$

where $\mathrm{E}(x_0)$ is the mean value of $x_0$ based on prior knowledge. Because the monitored system is possibly subject to changes, the predicted state $x_k^0$ based on $\theta^0$ may be biased. Assume that

$$x_k = x_k^0 + \varepsilon x_k^1 \tag{15}$$

where $x_k^1 \in \mathbb{R}^n$ is an unknown vector such that $\varepsilon x_k^1$ corresponds to the small bias caused by the small parameter increment $\tilde{\theta} = \varepsilon\theta^1$.

In the state equation (6a), substitute $A(\theta)$ with (13a), then

$$x_{k+1} = A^0 x_k + \varepsilon A^1 x_k + w_k$$

In the term $\varepsilon A^1 x_k$, replace $x_k$ with (15), then

$$x_{k+1} = A^0 x_k + \varepsilon A^1 x_k^0 + \varepsilon^2 A^1 x_k^1 + w_k$$

Omit the term involving $\varepsilon^2$ (because $\varepsilon$ is small), it then yields

$$x_{k+1} \approx A^0 x_k + \varepsilon A^1 x_k^0 + w_k. \tag{16}$$

Remind that $A^1 = A(\theta^1)$ is linearly parametrized by $\theta^1$, with $A(\cdot)$ as defined in (5). To make explicit the linear dependance on $\theta^1$ of the term $\varepsilon A^1 x_k^0$, divide $x_k^0$ into two sub-vectors $\bar{x}_k^0 \in \mathbb{R}^m$ and $\underline{x}_k^0 \in \mathbb{R}^m$ such that

$$x_k^0 = \begin{bmatrix} \bar{x}_k^0 \\ \underline{x}_k^0 \end{bmatrix},$$

then

$$\varepsilon A^1 x_k^0 = \varepsilon A(\theta^1) x_k^0 = \varepsilon\, \Psi_k \theta^1$$

with

$$\Psi_k \triangleq \begin{bmatrix} \mathrm{diag}(\bar{x}_k^0) & \mathrm{diag}(\underline{x}_k^0) & 0_{m\times nr} \\ \mathrm{diag}(\underline{x}_k^0) & -\mathrm{diag}(\bar{x}_k^0) & 0_{m\times nr} \end{bmatrix}, \tag{17}$$

following from (5a). The (approximate) equation (16) is then rewritten as

$$x_{k+1} \approx A^0 x_k + \Psi_k \tilde{\theta} + w_k \tag{18}$$

with the parameter increment $\tilde{\theta}$ as defined in (10).

In equation (18), the matrix $\Psi_k$ is composed of sub-vectors of the estimated state $x_k^0$, which is computed with the Kalman filter (14), hence $\tilde{\theta}$ is the only unknown in the term $\Psi_k\tilde{\theta}$. Therefore, the parameter increment $\tilde{\theta}$ appears *additively* in the state equation (18), in contrast to its *multiplicative* form in (6a).

The term $C(\theta)x_k$ in (6b) can be addressed similarly, thus

$$\begin{aligned} y_k &= (C^0 + \varepsilon C^1)x_k + v_k \\ &= C^0 x_k + \varepsilon C^1 x_k + v_k \\ &= C^0 x_k + \varepsilon C^1(x_k^0 + \varepsilon x_k^1) + v_k \\ &\approx C^0 x_k + \varepsilon C^1 x_k^0 + v_k \end{aligned}$$

where again the term involving $\varepsilon^2$ was omitted.

In the last equation, since $C(\theta)$ is linearly parametrized as in (5b),

$$\varepsilon C^1 x_k^0 = \varepsilon C(\theta^1) x_k^0 = \varepsilon\, \Phi_k \theta^1 = \Phi_k \tilde{\theta}$$

with

$$\Phi_k \triangleq \begin{bmatrix} 0_{r\times n} & (x_k^0)^T \otimes I_r \end{bmatrix}. \tag{19}$$

and $\tilde{\theta}$ as defined in (10).

To summarize, the initially considered *multiplicative* change detection problem formulated with the state space model (6) is now transformed to the detection of *additive changes* in the new state space model

$$x_{k+1} \approx A^0 x_k + \Psi_k \tilde{\theta} + w_k \tag{20a}$$
$$y_k \approx C^0 x_k + \Phi_k \tilde{\theta} + v_k \tag{20b}$$

where $y_k$ is the currently measured output, $A^0, C^0$ are known from the nominal model (typically obtained by means of system identification), $\Psi_k, \Phi_k$ are composed of $x_k^0$ which is estimated from the currently measured output $y_k$ through the Kalman filter (14).

## 4. ADDITIVE CHANGE DETECTION

Additive change detection problems in the form of (20) have been studied in (Zhang and Basseville [2014]), by transforming the dynamic system model (20) into an equivalent linear regression model through a particular Kalman filtering.

Apply the Kalman filter to the state space system (20) *while assuming* $\tilde{\theta} = 0$, it yields the one-step-ahead state prediction $x_{k|k-1}(\theta^0)$. Denote the innovation sequence (the prediction error) of this Kalman filter as

$$\zeta_k \triangleq y_k - C^0 x_{k|k-1}. \tag{21}$$

If $\tilde{\theta} = 0$, (actually no eigenstructure parameter change), then it is well known that the innovation sequence $\zeta_k$ is a centered Gaussian white noise.

However, if $\tilde{\theta} \neq 0$, the innovation sequence $\zeta_k$ is biased, because it is computed with the Kalman filter assuming $\tilde{\theta} = 0$. In this case, according to the Proposition 2 of (Zhang and Basseville [2014]), the innovation sequence $\zeta_k$ satisfies (the "$\approx$" would be replaced by "$=$" if the equalities in (20) were accurate)

$$\zeta_k \approx (C^0\Gamma_k + \Phi_k)\tilde{\theta} + e_k \tag{22}$$

where $\Gamma_k$ is recursively computed as

$$\Gamma_{k+1} = A^0(I_{n\times n} - KC^0)\Gamma_k + \Psi_k - A^0 K \Phi_k \tag{23a}$$
$$\Gamma_0 = 0, \tag{23b}$$

and $e_k$ is a white Gaussian noise of zero mean. The covariance matrix of $e_k$ is

$$\Sigma = C^0 P (C^0)^T + R \tag{24}$$

where $P$ is the solution of the algebraic Riccati equation associated to the Kalman filter and $R$ is the covariance matrix of the output noise $v_k$.

It turns out that this Kalman filter delivering the state prediction $x_{k|k-1}$ and the associated innovation sequence $\zeta_k$ in (21) is exactly the same as the one expressed in (14), as both are based on the same nominal system matrices $A^0, C^0$, the same noise covariance matrices, and the same measured output $y_k$. It means that $\zeta_k$ was already computed with (14), thus there is no need to run the Kalman filter twice.

In the algebraic equation (22), $\zeta_k$ is computed through (21) with the Kalman filter (14), $C^0 = C(\theta^0)$ is known from the nominal model, $\Gamma_k$ is computed through (23), $\Phi_k$ is expressed in (19), hence the parameter increment $\tilde{\theta}$ is the only unknown, apart from the white Gaussian noise $e_k$. In this *linear Gaussian framework*, it is well known that the generalized likelihood ratio (GLR) test (Basseville and Nikiforov [1993]) for $\tilde{\theta} \neq 0$ against $\tilde{\theta} = 0$ amounts to

$$\Omega = \sum_{k=1}^{N} (C^0\Gamma_k + \Phi_k)^T \Sigma^{-1} (C^0\Gamma_k + \Phi_k) \tag{25a}$$

$$\beta = \sum_{k=1}^{N} (C^0\Gamma_k + \Phi_k)^T \Sigma^{-1} \zeta_k \tag{25b}$$

$$s = \beta^T \Omega^{-1} \beta. \tag{25c}$$

The resulting statistics $s$ follows a $\chi^2$ distribution of $\dim(\tilde{\theta}) = n + nr$ degrees of freedom, central if $\tilde{\theta} = 0$, otherwise non-central with its non-centrality parameter equal to $\tilde{\theta}^T \Omega \tilde{\theta}$. The decision for change detection is thus made by comparing $s$ to a positive threshold.

## 5. ALGORITHMIC SUMMARY

The computational steps for the derived fault detection method are as follows.

In the *nominal reference state*:

(1) Obtain the nominal model parameter $\theta^0$ in (7), the associated system matrices $A^0$ and $C^0$ in canonical form as in (5a), and the noise covariances $Q$, $R$ and $S$ in (8). Usually, these quantities are estimated from system identification, e.g., using stochastic subspace identification with the UPC algorithm [Van Overschee and De Moor, 1996].
(2) Compute the nominal Kalman gain $K$ and the innovation covariance $\Sigma$ in (24).

To test if data $\{y_k\}_{k=1,\ldots,N}$ of the *current system* correspond to the nominal model or not, the test statistics $s$ is computed as follows:

(1) Apply the nominal Kalman filter to the data, obtaining the one-step-ahead state predictions $x_k^0$ in (14) and the innovations $\zeta_k$ in (21).
(2) Compute the matrices $\Psi_k$ in (17) and $\Phi_k$ in (19), and subsequently $\Gamma_k$ in (23).
(3) Compute the test statistics $s$ in (25).

To decide between $\tilde{\theta} = 0$ (no eigenstructure parameter change) and $\tilde{\theta} \neq 0$, the test statistics $s$ is compared to a threshold. This threshold can be obtained empirically based on realizations of $s$ on data from the nominal state for a given type I error.

## 6. APPLICATIONS

### *6.1 Numerical example*

Vibration-based damage detection on a simulated mass-spring chain with eight elements (Fig. 1) is considered as a first application of the derived fault detection method. The matrices $\mathcal{M}$, $\mathcal{C}$ and $\mathcal{K}$ in (1) of the nominal structural model are defined based on the masses $m_1 = m_3 = m_5 = m_7 = 1, m_2 = m_4 = m_6 = m_8 = 2$, stiffnesses $k_1 = k_3 = k_5 = k_7 = 1000, k_2 = k_4 = k_6 = k_8 = 500$ and a damping ratio of 2% for all modes. Datasets containing output-only time series of accelerations with time step $\tau = 0.05\,$s are simulated for different structural states at the four sensor coordinates from white noise excitation at all structural elements. White measurement noise is added with a magnitude of 5% of each generated output signal.

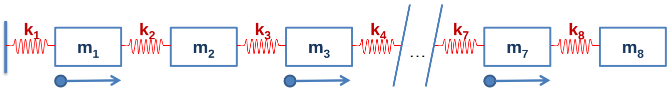


Fig. 1. Mass-spring chain with four sensors.

Stochastic subspace identification with the UPC algorithm [Van Overschee and De Moor, 1996] is used on a simulated

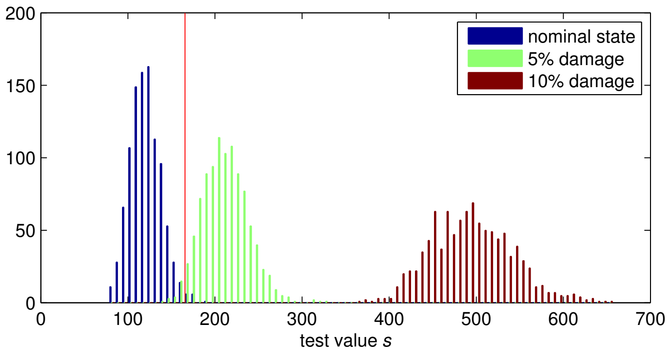


Fig. 2. Histogram of test values for the mass-spring chain in the nominal state, and 5% and 10% damage in spring 2.

dataset of length $N = 200{,}000$ to obtain the nominal model parameters. Three different structural states are considered for monitoring: the nominal state and two faulty (damaged) states with 5% and 10% stiffness decrease in spring 2, respectively. For each structural state, datasets of length $N = 10{,}000$ are simulated and the test statistics $s$ is computed. The test values of the nominal state are used to set up an empirical threshold to decide between $\tilde{\theta} \neq 0$ against $\tilde{\theta} = 0$. The resulting test values for 1000 datasets are shown in the histogram in Fig. 2, where a threshold (red line) is drawn from the nominal state for a 1% type I error. At this type I error, the power of the test for the 5% damage is 97%, and for the 10% damage the power of the test is 100%.

### *6.2 Lab experiment on a beam*

Experiments on a PVC beam were carried out by Brüel & Kjær. The structure's dimensions are 50 cm × 8 cm × 1 cm, being fixed on one side (Fig. 3). Damage is introduced in the beam by drilling small holes. Two damaged states with three holes and with five holes, respectively, are considered. For both the nominal and the damaged states, acceleration datasets of length $N = 295{,}936$ with a sampling frequency of 8192 Hz were recorded under white noise excitation by a shaker. The output data from the nine horizontal sensors on the top of the beam are used in this application. The data are downsampled and decimated by factor 6 to focus on the frequency range of interest, where the biggest damage led to less than 5% decrease in the structure's natural frequencies [Marin et al., 2015].

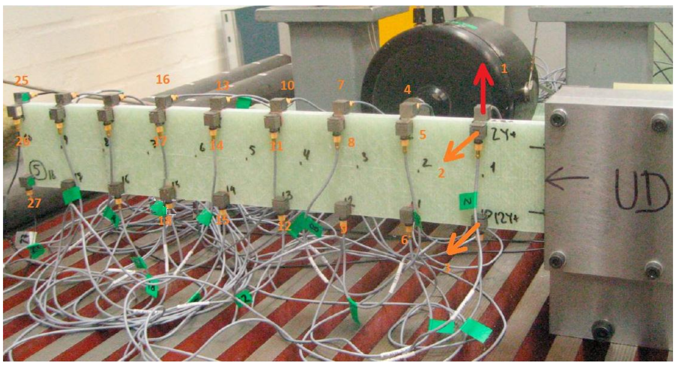


Fig. 3. Experimental setup of the beam.

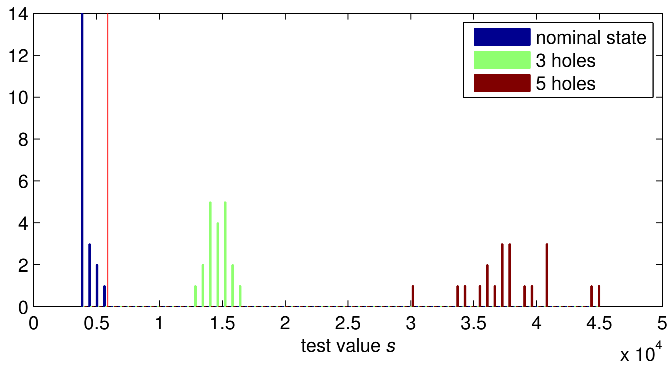


Fig. 4. Histogram of test values for the beam in the nominal state, and with three and five holes.

Using the first half of the dataset from the nominal state, the nominal model parameters are obtained from stochastic subspace identification. The available data of the nominal and the two damaged states are divided into 20 datasets for each state. Then, the test statistics $s$ is computed on each of these datasets. The resulting test values are shown in the histogram in Fig. 4. The test values from the nominal and damaged states are clearly separated, showing a successful detection in all cases. The increase of the test's non-centrality parameter with a bigger damage is also visible.

## 7. CONCLUSIONS

In this work, we have derived a test for the detection of small changes in the eigenstructure of a linear state-space system with application to vibration monitoring of mechanical or civil structures. The test is set up on variables that are easily estimated from system identification and the Kalman filter in the nominal (reference) state of the monitored structure. After transformations of the original detection problem, the standard GLR test is applied in the framework of a simple linear regression subject to additive white Gaussian noises. The normalization of the GLR test taking into account random uncertainties is solely based on the estimated state and output noise covariance matrices, which are standard results of the system identification procedure establishing the nominal model of the monitored structure.

The simple linear regression form of the transformed eigenstructure change detection problem makes possible the implementation of efficient algorithms for on-line monitoring. Moreover, when completed with a finite element model of the monitored structure expressed in a physical parameterization, as e.g. in [Döhler et al., 2014b], it is possible to determine the physical parameter(s) responsible for a detected damage. These topics will be the subject of future works.

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